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Primordial Gaussian fluctuations from cosmic defects

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Abstract

We extend our recent work on “two-metric” theories of gravity by showing how in such models cosmic defects can produce a spectrum of primordial Gaussian density perturbations. This will happen when the speed characterising the decay products of the defect network is much larger than the speed characterising gravity and all standard model particles. This model will exactly mimic all the standard predictions of inflationary models, and the only way of distinguishing the two will be via the detection of the decay products of the network.

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1. Introduction

The recent high-resolution measurements of Cosmic Microwave Background (CMB) anisotropies on degree and subdegree scales [1] have started to provide cosmologists with a realistic chance of probing the main physical mechanisms and conditions of the early universe [2]. However, one too often forgets that any subsequent treatment of the data (e.g., to determine “preferred” cosmological parameters) will require model-dependent assumptions, and one must make sure that these are properly justified. Two recent papers [3] illustrate a rather eclectic range of ways in which to use a given data set.

Consider the two basic paradigms that could be responsible for producing these anisotropies — topolog-

ical defect [4] and inflationary [5] models. There are of course some rather generic differences between the two, but still the task of unambiguously distinguishing between them is not at all trivial [6,7]. The presence of super-horizon perturbations or of “Doppler peaks” [8] on small angular scales, for example, are not good discriminants [9] (at least if they are taken on their own). This issue is further complicated since one can easily obtain “natural” models where both defects and inflation generate density fluctuations [10,11].

In previous work [12], we have presented an explicit example of a mechanism whereby the primordial fluctuations are generated by a network of cosmic defects, but are nevertheless very similar to a standard inflationary model. Such models arise in the context of “two-metric” theories [13–15]. The only difference between the observational consequences of these models and those of the standard inflationary scenario is a relatively small non-Gaussian component. Here we

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discuss an alternative model arising in the same context, but where the defect-induced primordial fluctuations are also Gaussian. In this case the evolution of the defect network is standard, but we require that the characteristic speed of the decay products of the defect network is much larger than the speed characterizing gravity and all the standard model particles. We show that this model will exactly reproduce the CMB and large-scale structure (LSS) predictions of the standard inflationary models, and the only way to identify it would be through the decay products of the defect network involved.

2. The model

As in our previous work [12], we shall consider the so-called “two-metric” theories [13–15], which contain two natural speed parameters. In [12] we assumed that the scalar field which produced the defects had a characteristic speed c_ϕ that was much larger than the characteristic speed of gravity and the standard model interactions. Here we shall instead concentrate on the decay products of the defect network.

In recent years there has been some debate over the issue of which is the dominant energy loss mechanism for defect networks. In particular, for the case of cosmic strings, there are two quite distinct possibilities, namely, an energy loss mechanism based on gravitational radiation [4] and one based on particle production [16,17]. Furthermore, some preliminary studies [18,19] indicate that the cosmological consequences — say, as measured from the CMB and matter power spectra — have some significant differences, although one should be cautious given the number of simplifying assumptions made in order to derive these results.

In this Letter we shall assume a two-metric theory where cosmic strings are produced, and where the long string network evolves in the standard way (with the comoving correlation length of the network being of the order of $c\eta$), and suppose that this network decays via particle production. Furthermore, we also assume that the decay products have a characteristic speed c_p which obeys $c_p \gg c$. In this case, the point will be to have the “compensation” scale much larger than the horizon size (defined in the usual way) — note that this will be so because one expects the compensation

scale to be of the order of the free streaming length of the decay products.

As in [12], the point of this paper is to describe a rather simple and general mechanism and its basic consequences, rather than a specific realization of it. Hence we will not concern ourselves with discussing particular models. In particular, the mechanism we will be discussing is independent of c_p being a constant or a time-varying quantity. For the time being we shall assume that c_p is a time-independent quantity, simply for the purpose of simplifying the discussion. We will later relax this assumption and discuss the implications for structure formation of variable c_p . Moreover, what the ultimate decay products of the defect network *are* will also not affect the model’s ability to work, although it will of course affect its detailed quantitative predictions (or, if one wants to put it in other words, its efficiency).

For example, the decay products could be massless particles with velocity $c_p \gg c$; these would, in analogy with the Cerenkov effect, emit gravitons and thus produce a stochastic gravitational wave background [15]. On the other hand, if the decay products are massive particles produced with characteristic velocity c_p , then we expect them to eventually be slowed down to below the characteristic speed of gravity, on a timescale related to the graviton emission rate. As will become clear below, our mechanism will be more efficient in the former case than in the latter. On the other hand, the subsequent evolution of these decay products can potentially be used to impose constraints on specific realizations of the mechanism, and the detection of a “background” of these decay products would provide an obvious way to test the mechanism.

The only detailed assumption we require is that the decay of the defect network proceeds in such a way to allow its evolution to be *qualitatively analogous* to the standard case [4,20–22]. In particular, we assume that some “scaling” solution will be reached after a relatively short transient period. However, we do allow (and indeed expect) a “scaling” solution that is *quantitatively different* from the usual, “scale-invariant” one — see [23] for a more detailed discussion of these concepts. This assumption still allows us to make use of some of the standard results on string-seeded structure formation in the following section.

We emphasize that there are significant differences between the model being discussed here and the

one previously presented in [12]. In our previous work the speed characterizing the defect-producing scalar field c_ϕ was much larger than the speed c characterizing gravity and standard model particles. Hence the comoving correlation length of the network was of order $c_\phi \eta \gg c\eta$. On the other hand, the decay products of the network were assumed to be standard, i.e., to have a characteristic speed c . In the present work we assume that the evolution of the network is standard (with the comoving correlation length of the network being of the order of $c\eta$), but that its decay mechanism is through a channel with some characteristic speed c_p that is much larger than the standard one.

3. Cosmological consequences

In the synchronous gauge, the linear evolution equations for radiation and cold dark matter perturbations, δ_r and δ_m , in a flat universe with zero cosmological constant are

$$\ddot{\delta}_m + \frac{\dot{a}}{a} \dot{\delta}_m - \frac{3}{2} \left(\frac{\dot{a}}{a} \right)^2 \left(\frac{a\delta_m + 2a_{\text{eq}}\delta_r}{a + a_{\text{eq}}} \right) = 4\pi G \Theta_+, \quad (1)$$

$$\ddot{\delta}_r - \frac{1}{3} \nabla^2 \delta_r - \frac{4}{3} \ddot{\delta}_m = 0, \quad (2)$$

where $\Theta_{\alpha\beta}$ is the energy–momentum tensor of the external source, $\Theta_+ = \Theta_{00} + \Theta_{ii}$, a is the scale factor, “eq” denotes the epoch of radiation-matter equality, and a dot represents a derivative with respect to conformal time. We will consider the growth of super-horizon perturbations with $ck\eta \ll 1$. Then Eq. (1) becomes:

$$\ddot{\delta}_m + \frac{\dot{a}}{a} \dot{\delta}_m - \frac{1}{2} \left(\frac{\dot{a}}{a} \right)^2 \left(\frac{3a + 8a_{\text{eq}}}{a + a_{\text{eq}}} \right) \delta_m = 4\pi G \Theta_+, \quad (3)$$

and $\delta_r = 4\delta_m/3$. Its solution, with initial conditions $\delta_m = 0$, $\dot{\delta}_m = 0$ can be written as

$$\delta_m^S(\mathbf{x}, \eta) = 4\pi G \int_{\eta_i}^{\eta} d\eta' \int d^3x' \mathcal{G}(X; \eta, \eta') \Theta_+(\mathbf{x}', \eta'), \quad (4)$$

$$\mathcal{G}(X; \eta, \eta') = \frac{1}{2\pi^2} \int_0^\infty \tilde{\mathcal{G}}(k; \eta, \eta') \frac{\sin kX}{kX} k^2 dk. \quad (5)$$

Here $X = |\mathbf{x} - \mathbf{x}'|$ and ‘S’ indicates that these are the “subsequent” fluctuations, according to the notation of [24], to be distinguished from “initial” ones.

We are interested in computing the inhomogeneities at late times in the matter era. When $\eta_0 \gg \eta_{\text{eq}}$, the Green functions are dominated by the growing mode, $\propto a_0/a_{\text{eq}}$, so the function we would like to solve for is [24]

$$T(k; \eta) = \lim_{\eta_0/\eta_{\text{eq}} \rightarrow \infty} \frac{a_{\text{eq}}}{a_0} \tilde{\mathcal{G}}(k, \eta_0, \eta). \quad (6)$$

Consider the growth of super-horizon perturbations generated during the radiation era, for which the transfer function can be written¹ [24]

$$T(0; \eta) = \frac{\eta_{\text{eq}}}{10(3 - 2\sqrt{2})\eta}. \quad (7)$$

The linear perturbations induced by defects such as cosmic strings, are the sum of initial and subsequent perturbations:

$$\begin{aligned} \delta_m(k; \eta_0) &= \delta_m^I(k; \eta_0) + \delta_m^S(k; \eta_0) \\ &= 4\pi G(1 + z_{\text{eq}}) \int_{\eta_i}^{\eta_0} d\eta T_c(k; \eta) \tilde{\Theta}_+(k; \eta), \end{aligned} \quad (8)$$

where η_i is the time when the network of cosmic defects was generated and T_c is the transfer function for the subsequent perturbations, those generated actively by the defects. In order to include compensation for the initial perturbations, δ_m^I , the substitution is usually made:

$$T_c(k; \eta) = (1 + (k_c/k)^2)^{-1} T(k; \eta), \quad (9)$$

where k_c is a long-wavelength cut-off at the compensation scale. In the case where the defects have several possible decay channels² there may be several scales associated with compensation due to the different dynamics of the defect decay products. However,

¹ For super-horizon perturbations generated during the matter era, the transfer function would differ from the above by a factor of two.

² For the case of cosmic strings these could be loops, gravitational radiation and in our theory particles with velocity $c_p \gg c$.

for simplicity we shall assume that Eq. (9) is a good approximation with the compensation scale, k_c , determined by particles generated by the defect network with $c_p \gg c$. Consequently, we expect $k_c \sim (c_p \eta)^{-1}$. If there are other decay channels this assumption may slightly alter the power spectrum normalization but will not otherwise affect the predictions of our model.

Note also that both in the present work and in our previous work [12] we get a compensation scale that is much larger than $c\eta$, but the reasons for this are slightly different in the two cases. In [12] the compensation scale is expected to be of the order of the correlation length of the network, while in the present work the compensation scale is expected to be of the order of the free-streaming length of the decay products.

For $(c_p \eta_0)^{-1} \ll k \ll (c_p \eta_i)^{-1}$ the analytic expression for the power spectrum of density perturbations induced by defects can be written as

$$P(k) = 16\pi^2 G^2 (1 + z_{\text{eq}})^2 \int_0^\infty d\eta \mathcal{F}(k, \eta) |T_c(k, \eta)|^2, \quad (10)$$

where $\mathcal{F}(k, \eta)$ is the structure function which can be obtained directly from the unequal time correlators [25,26]. Still in the case of super-horizon perturbations, it can easily be shown [25] that for a scaling network $\mathcal{F}(k, \eta) = \mathcal{F}(k\eta)$ which, combined with Eqs. (7), (9) and (10), gives

$$P(k) \propto \int_0^\infty d\eta \mathcal{S}(k\eta) / \eta^2 \propto k \quad (11)$$

for super-horizon modes. Here the function \mathcal{S} , is just the structure function, \mathcal{F} , times the compensation cut-off function.

Up until now we only considered the spectrum of primordial fluctuations induced by cosmic defects (by primordial we mean generated at very early times). In our model a Harrison–Zel’dovich spectrum is predicted (see Eq. (11)) just as in the simplest inflationary models. The final processed spectrum taking into account the growth of the perturbations inside the horizon in the radiation and matter eras will also be the same as for the simplest inflationary models. Note that if c_p is a time varying quantity then the compensation cut-off is no longer a function of $k\eta$

alone, and so it is possible to have deviations from a pure Harrison–Zel’dovich spectrum just as in generic inflationary models.

On large scales $k \ll (c\eta)^{-1}$ the structure function $\mathcal{F}(k, \eta)$ has a white noise spectrum. The turn-over scale, if it exists, only appears at the correlation length of the network $k_\xi \gtrsim (c\eta)^{-1}$ [25,27]. This means that perturbations induced on scales larger than the correlation length are generated by many defect elements and, therefore, have a Gaussian distribution according to the central limit theorem. On the other hand, perturbations induced on smaller scales are very non-Gaussian because they can be either very large within the regions where a string has passed by or else very small outside these. This allows us to roughly divide the power spectrum of cosmic-string-seeded density perturbations into a nearly Gaussian component generated when the string correlation length was smaller than the scale under consideration, and a strongly skewed non-Gaussian component generated when the string correlation length was larger (we call these the “Gaussian” and “non-Gaussian” contributions, respectively).

The ratio of these two components may be easily computed by splitting the structure function in (10), in two parts: a Gaussian part $\mathcal{F}_g(k, \eta) = \mathcal{F}(k, \eta)$ for $k < k_\xi$ ($\mathcal{F}_g = 0$ for $k > k_\xi$) and a non-Gaussian part $\mathcal{F}_{ng}(k, \eta) = \mathcal{F}(k, \eta)$ for $k > k_\xi$ ($\mathcal{F}_{ng} = 0$ for $k < k_\xi$). We can then integrate (10) with this Gaussian/non-Gaussian split, to compute the relative contributions to the total power spectrum. The final result will of course depend on the choice of compensation scale k_c , but recall that in any case we expect the network correlation length to be much smaller than the compensation scale. Thus, given that in our model $k_c \ll (c\eta)^{-1}$ the “non-Gaussian” component will simply be too weak to be detected.

By allowing for a characteristic velocity for one of the decay channels of a defect network c_p much larger than the velocity of light (and gravity) c we were able to construct a model with primordial, adiabatic ($\delta_r = 4\delta_m/3$), nearly Gaussian fluctuations whose primordial spectrum is of the Harrison–Zel’dovich form. This model is indistinguishable from the simplest inflationary models (as far as structure formation is concerned). The C_1 spectrum and the polarization curves of the CMBR predicted by this model should also be identical to the ones predicted in the simplest inflation-

ary models as the perturbations in the CMB are not generated directly by the defects. Moreover, the gravitational wave background generated by the defects in this theory should be too weak to be detected as the energy scale of the defects can be significantly lower than in the standard case.

4. Discussion and conclusions

Following up our previous work on the cosmological consequences of “varying speed of light” theories [12,28,29], we have presented a further general illustration of non-negligible overlap between topological defect and inflationary structure formation models, in the context of “two-metric” theories of gravity.

According to Liddle’s criterion [6], such “mimic” models of inflation require some form of “violation of causality”. In our previous work [12], this was provided by a defect-producing scalar field with a characteristic speed much larger than that of gravity and the standard model interactions. In the present work, it is instead provided by a similarly large “superluminal” characteristic speed of the decay products of the defect network. Hence we see that defects in two-metric theories can produce either Gaussian or non-Gaussian [12] primordial fluctuations. The only distinguishing characteristics of these models, by comparison with the simplest inflationary models, will be a small non-Gaussian signal in the former case, and a background of the defect decay products in the latter. Both of these could be detected by future experiments. A more detailed discussion will be presented in [30].

As we already pointed out in [12], these models might admittedly seem somewhat “unnatural” in the context of our present theoretical prejudices, though they are certainly not the only ones to fit in this category [9,31]. However, if one keeps in mind that any fully consistent cosmological structure formation model candidate should eventually be derivable from fundamental physics, one could argue that at this stage they are *ceteris paribus*, on the same footing as inflation. Certainly no single fully consistent realization of an inflationary model is known at present.

In any case, what these models do provide is explicit evidence of the fact that one must be extremely careful with one’s prior assumptions when using cosmological datasets, and that one must keep looking for efficient

and unambiguous ways to test the main paradigms of cosmology. We shall return to this topic in a future publication.

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